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TWO PHOTON PROCESSES AND EFFECTIVE LAGRANGIANS WITH AN EXTENDED SCALAR SECTOR

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ABSTRACT

We consider the Standard Model with an extended scalar sector, and study the possible effects of the physics underlying such a model using an effective lagrangian parametrization. It is found that certain two photon processes offer windows where such heavy interactions might be glimpsed, but the realization of this expectation requires enormous experimental precision.

1. Introduction

The use of effective lagrangians¹ in parametrizing physics beyond a given theory has been studied extensively in the recent literature, especially within the context of the Standard Model.² The formalism generates a model-independent parametrization of the low energy limit of any Green function in terms of a set of unknown constants, each of which multiplies a local operator involving only the low energy fields; all symmetries are also naturally preserved. Despite the fact that the formalism involves, in principle, an infinite number of local operators in the light fields, only a finite number of them need to be considered in any given calculation; the number of operators which are considered is determined by the required degree of accuracy: for higher precision more terms in the effective lagrangian must be included, and the number of parameters increases. Any two models of describing the heavy physics will generate low energy Green functions whose expressions fit the effective lagrangian parametrization (the explicit values of the parameters are, of course, model dependent).

The requirement that the low energy particle content should be the same as in the Standard Model is clearly an assumption. One can envisage, for example, the case where the Higgs particle is either heavy or absent [4], or a situation where there is an additional relatively light vector boson [5]. In this paper we will consider a different modification of the low energy lagrangian for which the low energy scalar sector contains two doublets. Such a model, motivated by various supersymmetric extensions of the Standard Model [6], will be our low energy theory; the adjective “light” will refer to it.

Starting from this low energy theory we will investigate the possible effects of (heavy) physics underlying it. The purpose of this investigation is dual: on the one hand we will determine the difficulties in uncovering heavy physics effects even in some very favorable circumstances, which will be of use when and if the minimal supersymmetric extension of the Standard Model is verified experimentally. On the other hand the calculations presented in this paper provide a good illustration of the uses of effective operators within the context of renormalization theory.

Since the light theory is renormalizable and has light scalar excitations (whose masses are not protected against $O(\Lambda)$ radiative corrections), the underlying physics is expected to be light and decoupling [3]. We will denote by Λ the scale at which the underlying physics becomes apparent, then all observables can be expanded in a power series in $1/\Lambda$ [7].^{#1}

The two doublet model is known to have problems with flavor changing neutral currents. We chose to alleviate these difficulties by imposing the usual discrete symmetry [8]. We will for simplicity also impose this symmetry on the heavy operators (within the framework of this paper this means that the processes which violate this symmetry occur at a scale $\Lambda' \gg \Lambda$, which is supported by explicit computation of the bounds on the scale of process which mediate flavor changing neutral currents [9]). We will argue below that the same estimates of the sensitivity of a given experiment to Λ are obtained even when the underlying physics does not obey the discrete symmetry. We will also ignore CP violation in the light theory;

^{#1} Note that in the process of integrating out the heavy physics some quantities will get corrections which do not vanish as $\Lambda \rightarrow \infty$, but all these contributions can be absorbed in a renormalization of the two doublet model parameters and are therefore unobservable. The effects of these contributions affect the naturalness of the model and will not be considered further in this paper.

more precisely, we will assume that the scale of CP violation is $\gtrsim \Lambda$, as discussed in section 4.

A general study of the two doublet model together with all operators of dimension > 4 would be both prohibitive and obscure. Fortunately this is not necessary: most processes receive light physics contributions (*i.e.*, those generated by the two-doublet extension of the Standard Model) which dominate any contribution from the heavy physics. Thus we need only concentrate on those processes which are suppressed within the two-doublet model and which acquire significant contributions due to the inclusion of effective operators.

As an example of the above situation we will study the two photon decay of the CP-odd scalar a_0 present in this theory, as well as a_0 production in polarized photon colliders. Since this particle has no couplings to the gauge bosons within the light theory, the usual contribution to these process are mediated by fermion loops. Explicit calculation as well as simple arguments show that these graphs are sizable only for the top quark contribution which is proportional to $^{\#2} \cot \beta$ and will be small in the $\beta \rightarrow \pi/2$ limit. Alternatively we will see that the production of the a_0 within the light theory in photon collisions (via quark loops) is suppressed for a certain combination of photon polarizations. This, however, is not the case for the amplitude induced by the effective operators. We can then arrange for a suppression of the light-theory production amplitude, with the corresponding enhancement of the “anomalous” signal, by choosing the photon polarizations

$^{\#2} \tan \beta$ denotes the ratio of the vacuum expectation values, see below.

appropriately.^{#3}

The organization of the paper is as follows. In the next section we will construct the relevant operators for the above processes. The calculation of the corresponding amplitudes together with some illustrative numerical results is presented in section 3. Section 4 contains some parting comments. Details of the calculations are summarized in the appendix.

2. THE LAGRANGIAN

We begin with a brief description of the model starting with the renormalizable interactions; later we turn to the study of heavy physics effects using effective operators.

The scalar sector contains two doublets ϕ_a ($a = 1, 2$), while the fermionic and gauge particle contents are identical to those in the Standard Model. As mentioned in the introduction we will impose a \mathbf{Z}_2 symmetry on the low energy sector in order to suppress flavor changing neutral currents. Under this symmetry ϕ_1 and all right handed fermions are odd, while the remaining particles are even. We will use the conventions of Refs. 9 and 10.

The light scalar potential (that is, the terms in the potential of dimension ≤ 4)

^{#3} The constants multiplying the effective operators are estimated using standard arguments (see below); it is of course possible for some of these constants to be suppressed due to unknown effects, we will comment on this possibility later on.

is given by

$$\begin{aligned}
V = & \lambda_1 \left(\phi_1^\dagger \phi_1 - v_1^2 \right)^2 + \lambda_2 \left(\phi_2^\dagger \phi_2 - v_2^2 \right)^2 \\
& + \lambda_3 \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 - v^2 \right)^2 + \lambda_4 \left[\left(\phi_1^\dagger \phi_1 \right) \left(\phi_2^\dagger \phi_2 \right) - \left| \phi_1^\dagger \phi_2 \right|^2 \right] \\
& + \lambda_5 \left[\text{Re} \left(\phi_1^\dagger \phi_2 \right) - v_1 v_2 \right]^2 + \lambda_6 \left[\text{Im} \left(\phi_1^\dagger \phi_2 \right) \right]^2
\end{aligned} \tag{2.1}$$

In terms of the mass eigenstates the scalar fields are

$$\begin{aligned}
\phi_1^0 &= v_1 + \frac{1}{\sqrt{2}}(c_\alpha H_0 - s_\alpha h_0) + \frac{i}{\sqrt{2}}(c_\beta G_0 - s_\beta a_0) \\
\phi_2^0 &= v_2 + \frac{1}{\sqrt{2}}(s_\alpha H_0 + c_\alpha h_0) + \frac{i}{\sqrt{2}}(s_\beta G_0 + c_\beta a_0) \\
\phi_1^+ &= c_\beta G^+ - s_\beta H^+ \\
\phi_2^+ &= s_\beta G^+ + c_\beta H^+
\end{aligned} \tag{2.2}$$

where

$$\phi_a = \begin{pmatrix} \phi_a^+ \\ \phi_a^0 \end{pmatrix}, \quad a = 1, 2 \tag{2.3}$$

We used the usual definitions $\tan \beta = v_2/v_1$, $v^2 = v_1^2 + v_2^2$; $c_\alpha = \cos \alpha$, etc. The fields G_0 , G^+ and G^- correspond to the would-be Goldstone bosons (with $G^- = (G^+)^\dagger$), while the other scalar fields are physical.

In the calculations below we will adopt the following notation and conventions (see Ref. 9): ℓ and q denote the left-handed lepton and quark doublet respectively, e , u and d denote the corresponding right-handed $SU(2)$ singlets, (of particular interest to us will be the case where u denotes the right-handed top quark and q the left-handed top-bottom doublet). The $SU(2)$ gauge fields are denoted by W_μ^I (where I is a weak isospin index) and the $U(1)$ gauge field is labelled B_μ ,

the corresponding field strengths are^{#4} $W_{\mu\nu}^I$ and $B_{\mu\nu}$; finally τ^I denote the Pauli matrices and $\tilde{\phi}_k = i\tau^2\phi_k^*$. The covariant derivatives are $D_\mu = \partial_\mu - \frac{ig}{2}\tau^I W_\mu^I - \frac{ig'}{2}Y B_\mu$ where Y is the hypercharge and g, g' the gauge couplings corresponding to $SU(2)_L$ and $U(1)_Y$ respectively.

With these preliminaries we can now discuss the possible effects of heavy physics on low energy observables (where “low energy” refers to scales below that of symmetry breaking in the above two-doublet model), concentrating on the corresponding modifications to the coupling of the CP odd scalar a_0 to two photons. We will assume that the underlying physics is decoupling and weakly coupled, and denote by Λ its characteristic scale (the scale at which it becomes directly observable). We only consider the weakly coupled case since for strongly coupled underlying physics it is difficult to maintain the scalars ϕ_a naturally light (compared to Λ) without considerably modifying the low energy spectrum [3]. The decoupling theorem [7] then implies that all the effects of heavy excitations at low energies can be parametrized in a model independent manner by a series of local operators which satisfy the same symmetries as the low energy theory, and whose coefficients are suppressed by the appropriate power of Λ . It is of course possible for the underlying theory to have several mass scales, if this is the case Λ will denote the smallest of these scales.^{#5}

The construction of the effective lagrangian closely parallels the one described in Ref.9. The terms of dimension four are the ones in the two-doublet model; there are no terms of dimension five due to Lorentz and gauge symmetries (right-

#4 $W_{\mu\nu}^I$ is the full non-Abelian expression for the curvature.

#5 This implies that a suppression factor $\sim (\Lambda/\Lambda')^2$ may be present in some of the operators.

handed neutrinos are assumed to be absent). Therefore all the effects of the heavy excitations will be suppressed by at least two powers of Λ . The simplest way of obtaining these operators is to take the list provided in Ref.9 and replace the Standard Model doublet by either ϕ_1 or ϕ_2 ^{#6}.

Each of the effective operators appears in the effective lagrangian multiplied by an undetermined coefficient. This apparently implies that there is no way of extracting from this formalism quantitative estimates. Fortunately this is not the case: having taken the underlying physics to be weakly coupled, the order of magnitude of the coefficient of an operator is determined by the type of graph in the underlying theory which generates it. If an operator is generated at tree level by the heavy dynamics, it will appear with a coefficient $\sim h/\Lambda^2$ where h denotes a product of coupling constants in the underlying theory; these by assumption are $\lesssim 1$, so that we also expect $h \lesssim 1$. Loop-generated operators will acquire an extra suppression factor, so that their coefficient are of the form $h'/(4\pi\Lambda)^2$ with $h' \lesssim 1$. The additional loop factor $\sim 1/(4\pi)^2$ insures that the loop-generated operators are subdominant. We will assume that the heavy physics is described by a gauge theory and, for quantitative estimates will take the corresponding gauge coupling to be ~ 1 ; one should keep in mind that the results thus obtained represent an upper bound and could be suppressed by a significant amount.

The standard (two-doublet) contribution to the a_0 - γ - γ coupling occurs at one loop [10]. The only dimension six operators which generate tree-level contributions

^{#6} We have verified that this procedure generates all the operators relevant for the calculations below, despite the fact that the equations of motion were used in Ref. 9 to eliminate some operators.

to this vertex are

$$\begin{aligned}\mathcal{O}_{\text{tree1}} &= (B_{\mu\nu})^2 \mathbf{Im} \left(\phi_1^\dagger \phi_2 \right), & \tilde{\mathcal{O}}_{\text{tree1}} &= B_{\mu\nu} \tilde{B}_{\mu\nu} \mathbf{Im} \left(\phi_1^\dagger \phi_2 \right), \\ \mathcal{O}_{\text{tree2}} &= \left(W_{\mu\nu}^I \right)^2 \mathbf{Im} \left(\phi_1^\dagger \phi_2 \right), & \tilde{\mathcal{O}}_{\text{tree2}} &= W_{\mu\nu}^I \tilde{W}_{\mu\nu}^I \mathbf{Im} \left(\phi_1^\dagger \phi_2 \right); \end{aligned} \quad (2.4)$$

(together with their hermitian-conjugate counterparts), which are loop-generated [11]: there are no dimension six, tree-level-generated operators that contribute to the a_0 - γ - γ vertex at tree level. Note that these operators violate the \mathbf{Z}_2 symmetry. A consistent loop expansion then requires that we include all tree-level contributions containing one (2.4) insertion. We must also include all one-loop graphs containing one \mathcal{O} insertion, provided this operator is generated at tree level. The one loop graphs containing insertions of loop-generated operators can be ignored; this considerably reduces the number of terms that need to be considered.

At this point we can either assume that the underlying dynamics respects the discrete symmetry or not. The precise results differ depending on which of these cases is realized; note however that in either case all contributions to the vertex of interest occur at one loop; either through tree-level-generated operators in one loop graphs, or through loop-generated operators in tree level graphs. In this paper we are only interested in estimating the effects of the underlying physics and for this it is sufficient, in view of the previous comments, to consider the situation where the underlying physics obeys the discrete symmetry. This assumption has the added advantage of simplifying the calculations.

When the physics underlying the two doublet model is assumed to respect the discrete symmetry there are no tree-level contributions to the a_0 - γ - γ vertex since the operators in (2.4) are forbidden. It follows that the dimension six modifications

of the low energy lagrangian contribute to the processes under consideration only through (explicit) one loop graphs; as mentioned above only tree-level-generated operators need be considered.

Due to the absence of tree level interactions, the loop contributions must be *finite*. Indeed, any divergence can be associated with a local operator respecting the symmetries of the model; the effective lagrangian already contains all such operators, and all divergences can be absorbed in a renormalization of the effective lagrangian coefficients [12]. Since there is no tree level operator (when the \mathbf{Z}_2 symmetry is imposed) containing an a_0 - γ - γ term, it follows that the corresponding three point function must be finite, else it would be impossible to absorb its divergence. The cancellation of divergences, together with electromagnetic gauge invariance, provide important checks on the algebra. Imposing the discrete symmetry on the effective operators also implies that there is no dependence (to one loop) on the renormalization scale.

The effective lagrangian takes the form

$$\mathcal{L} = \mathcal{L}_{\text{two doublets}} + \frac{1}{\Lambda^2} \sum_i [y_i \mathcal{O}_i + \text{h. c.}] + O(1/\Lambda^3) \quad (2.5)$$

where the summation over i runs over the set of (tree-level-generated) operators given in the appendix. The unknown constants y_i depend on the underlying physics and, based on the discussion above, are expected to be ~ 1 .

The most important terms in (2.5) are generated by the operators

$$\mathcal{O}_{u\varphi;211} = \left(\phi_2^\dagger \phi_1\right) \left(\bar{q} u \tilde{\phi}_1\right), \quad \mathcal{O}_{ijkl}^{(1)} = \left[\partial_\mu \left(\phi_i^\dagger \phi_j\right)\right] \left[\phi_k^\dagger \overleftrightarrow{D}_\mu \phi_l\right]; \quad (2.6)$$

which dominate the heavy physics contributions to the a_0 - γ - γ vertex (in $\mathcal{O}^{(1)}$, $i, j, k, l =$

1, 2 and only those terms with an even number of ϕ_1 factors are considered due to the discrete symmetry). The corresponding contributions are not suppressed in the limit of large $\tan \beta$; nor are they suppressed by (explicit) small fermions masses.^{#7}.

3. Calculation of the amplitudes

Having described the lagrangian we can use it to evaluate the three point function for two photons and one a_0 . The photons will be labelled by the subscripts 1 and 2, the corresponding momenta and polarization vectors will be denoted by k_a and e_a ($a = 1, 2$) respectively. We will assume that all (external) particles are on shell.

Due to electromagnetic gauge invariance there are two possible tensorial structures allowed for the amplitude:

$$\mathcal{P}_1 = (k_1 \cdot k_2)(e_1 \cdot e_2) - (k_1 \cdot e_2)(k_2 \cdot e_1), \quad \mathcal{P}_2 = i\epsilon_{\alpha\beta\mu\nu}e_1^\alpha e_2^\beta k_1^\mu k_2^\nu \quad (3.1)$$

correspondingly the amplitude can be written as

$$\mathcal{A}_{a_0-\gamma-\gamma} = \mathcal{M}_1 \mathcal{P}_1 + \mathcal{M}_2 \mathcal{P}_2. \quad (3.2)$$

The light-physics amplitude generated via quark loops contains only \mathcal{P}_2 . In contrast the \mathcal{O} induced terms can be proportional to either quantity, in particular all bosonic loops give results proportional to \mathcal{P}_1 . This implies that the decay $a_0 \rightarrow$

^{#7} It is of course possible for the underlying dynamics to generate coefficients which are small for large $\tan \beta$ and/or contain small Yukawa couplings, we will not consider this possibility here.

$\gamma\gamma$ will have a modified width and angular distribution due to the presence of the operators \mathcal{O} . Unfortunately the branching ratio is so small that the modification of the angular distribution is unobservable (at least in the foreseeable future).

In contrast, the presence of terms proportional to \mathcal{P}_1 in the amplitude can have important effects in a_0 production in photon colliders. These machines will (probably) allow the possibility of polarizing the photons whence the terms $\propto \mathcal{P}_2$ can be suppressed by the appropriate choice of initial polarizations thus enhancing the heavy physics contributions.

The calculation of the amplitude involves both one particle irreducible and one particle reducible diagrams. The latter arise due to the $a_0 - h_0$ and $a_0 - H_0$ mixings induced by operators such as the second one in (2.6). The main contributions to the 1PI graphs come from the first operator in (2.6), which also becomes the overall dominant term when α is close to zero or to $\pi/2$ (provided $\tan \beta$ is large). For intermediate values of α the contributions from both operators in (2.6) are comparable.^{#8}

The total amplitude derived from (2.5) is

$$\mathcal{A}_{a_0 \rightarrow \gamma\gamma} = \mathcal{A}_{\text{light}} + \frac{1}{\Lambda^2} \sum_i y_i \mathcal{A}_i \quad (3.3)$$

where \mathcal{A}_i denotes the contribution to the a_0 - γ - γ (on-shell) three point function generated by \mathcal{O}_i (without the factor y_i/Λ^2). From $\mathcal{A}_{a_0 \rightarrow \gamma\gamma}$ the two-photon decay width of the a_0 or its production cross section in photon collisions can be evaluated

^{#8} The 1PR graphs will in general exhibit resonances at the mass of the h_0 and H_0 . The previous comments correspond to energies outside these regions.

directly. Note that the terms containing \mathcal{P}_1 and \mathcal{P}_2 will not interfere in polarization-averaged widths or cross-sections. The quantities \mathcal{A}_i are displayed in the appendix.

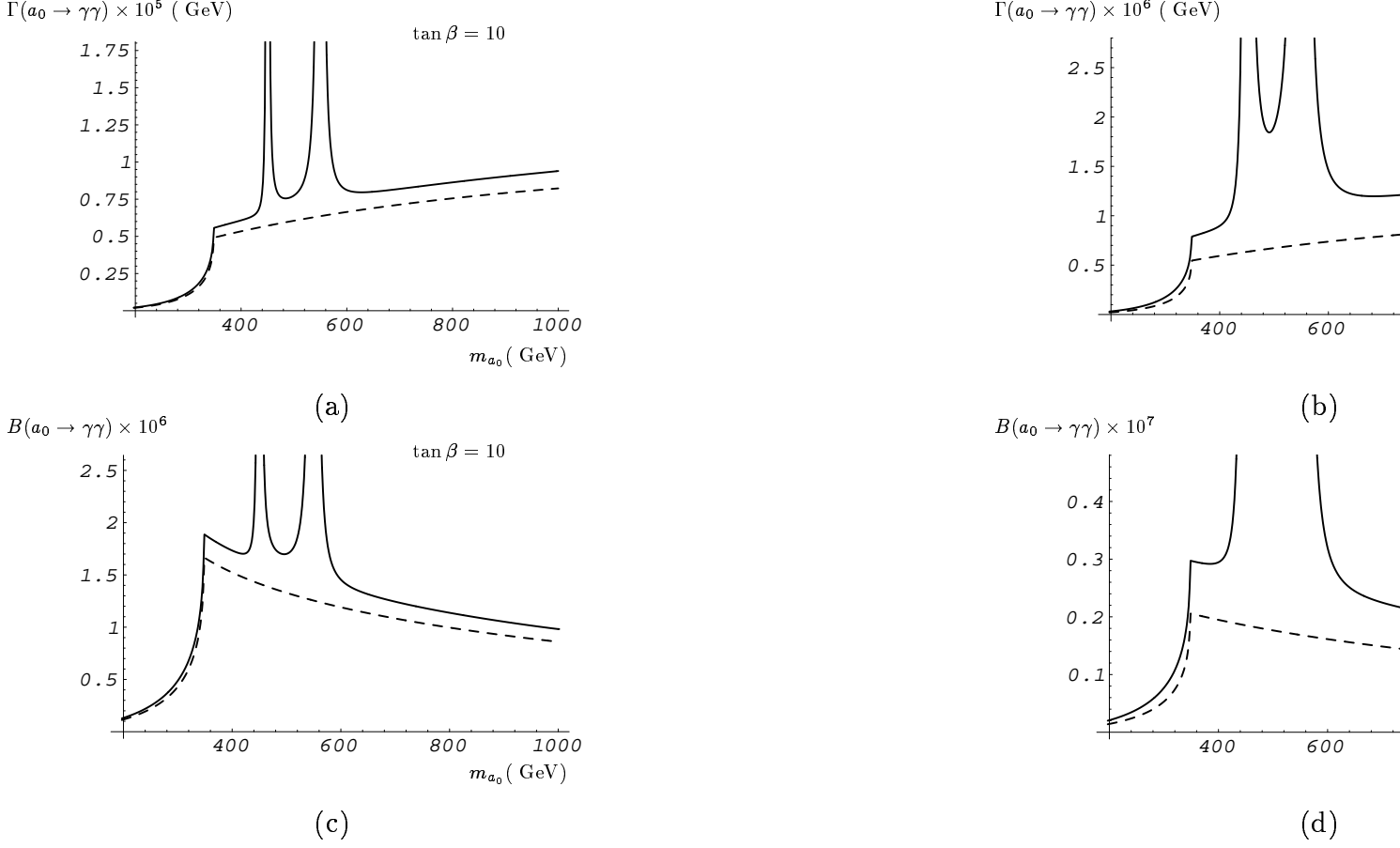


Figure 1. Width and branching ratios for the decay $a_0 \rightarrow \gamma\gamma$ as a function of m_{a_0} for two values of β . The solid curves correspond to the theory with effective operators for $\Lambda = 1 \text{ TeV}$, the dashed curves give the pure two-doublet model results. We chose $m_{h_0} = 450 \text{ GeV}$, $m_{H_0} = 550 \text{ GeV}$, $m_{H^\pm} = 500 \text{ GeV}$; $m_{\text{top}} = 174 \text{ GeV}$, $\alpha = 45^\circ$. The resonance peaks are produced by the 1PR diagrams.

For generic values of the parameters in the standard two-doublet lagrangian the contributions from the \mathcal{O}_i are small due to the presence of the $1/\Lambda^2$ factor. This suppression can be countered in two ways. For a_0 production we can use the photon polarization to suppress the light contribution and bring the anomalous effects to the foreground. For the two-photon decay of the a_0 the light contributions are proportional to either $m_b \tan \beta$ or $m_t \cot \beta$ and are suppressed when $|\beta - \pi/2|$ is

similar to zero (though larger than $\sim m_b^2/m_t^2$). The only terms which can compete with the light contributions are those which do not vanish as $\beta \rightarrow \pi/2$ and are not suppressed by a small mass factor. These constraints are satisfied only by the operator (2.6).

Using the results of the appendix we evaluated the $a_0 \rightarrow \gamma\gamma$ width and branching ratio for several illustrative choices of the masses and couplings, the results are presented in Fig. 1^{#9}.

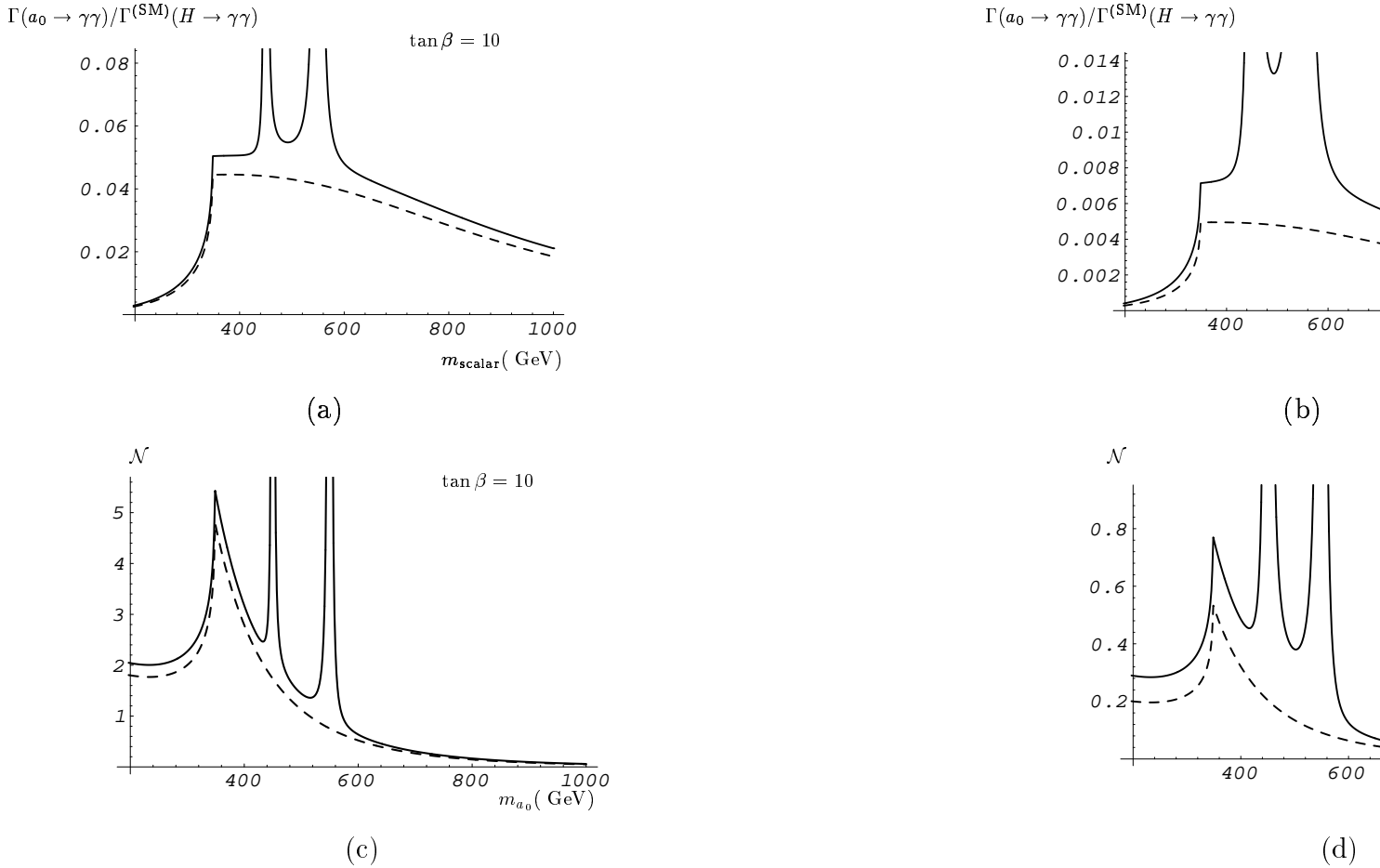


Figure 2. **(a),(b)**: Ratio of the total $a_0 \rightarrow \gamma\gamma$ decay width to the decay width of a Standard Model Higgs particle of the same mass into two photons. **(c),(d)** Total number of yearly events at the LHC (C.M. energy= 7TeV, luminosity 100/fb). The solid curves correspond to the theory with effective operators for $\Lambda = 1$ TeV, the dashed curves give the pure two-doublet model results. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^+} = 500$ GeV and $\Lambda = 1$ TeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$.

^{#9} These results are presented for the choice of constants y_i specified in the Appendix.

As can be seen from this figure there is a $\sim 50\%$ increase from the light-physics values for this decay width; despite this one has to face the complications produced by the smallness of the branching ratio, being, in the most favorable case $\sim 10^{-7}$ and about 3% of the two-photon width of a Standard Model Higgs boson of the same mass, as shown in Fig. 2a and 2b. The number of events, showed in 2c,d is marginal for one year of LHC running, but, mainly due to the resonance enhancement, deviations from the two doublet model would be observable for moderate values of m_{a_0} .

This implies that when (and if) the a_0 is discovered at the LHC, its branching ratio into two photons could prove a sensitive reaction in which to study physics beyond this excitation but the experimental sensitivity required is enormous. We have verified that all heavy physics effects are completely obscured when $\tan\beta$ is smaller than ~ 5 ; the results are also insensitive to the mass of the charged Higgs.

We also investigated the behaviour of the two-photon branching ratio as a function of Λ ; the results are presented in Fig. 3. Based on this graph we conclude that this reaction will be able to probe physics up to ~ 1 TeV.

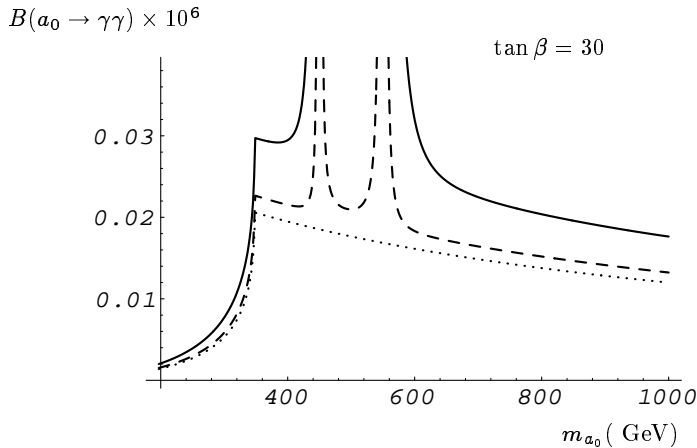


Figure 3. Branching ratio for the decay $a_0 \rightarrow \gamma\gamma$ for $\Lambda = 1$ TeV (solid curve), $\Lambda = 2$ TeV (dashed curve) and $\Lambda = \infty$ (dotted curve). We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^\pm} = 500$ GeV and $\tan\beta = 30$; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$.

Now we consider a_0 production in photon-photon collisions. From general considerations the complete amplitude is of the form $\mathcal{A}_{\gamma\gamma\rightarrow a_0} = \mathcal{M}_1\mathcal{P}_1 + \mathcal{M}_2\mathcal{P}_2$. We will assume that both initial photons have the same polarization matrix. In this case we obtain

$$|\mathcal{A}_{\gamma\gamma\rightarrow a_0}|^2 = \frac{s^2}{8} \left[|\mathcal{M}_1|^2 (1 + \boldsymbol{\xi}_- \cdot \boldsymbol{\xi}_+) + |\mathcal{M}_2|^2 (1 - \boldsymbol{\xi}_+^2) \right] \quad (3.4)$$

where $\boldsymbol{\xi}_\pm = (\xi_1, \pm\xi_2, \xi_3)$ and ξ_i are the Stokes parameters for the photons (assumed the same for both).

The ratio $(1 - \boldsymbol{\xi}_+^2)/(1 - \boldsymbol{\xi}_- \cdot \boldsymbol{\xi}_+)$ is independent of the polarization of the initial photons and depends only on the polarization of the initial electrons (for perfectly polarized electrons $\boldsymbol{\xi}_+^2 = 1$). As an illustration we will assume that the initial electrons are 70% longitudinally polarized, the electron and photon energies are taken equal to 500 GeV and 1.17 eV. In this case [13] $(1 - \boldsymbol{\xi}_+^2)/(1 - \boldsymbol{\xi}_- \cdot \boldsymbol{\xi}_+) \simeq 0.86$ (and becomes $\simeq 0.7$ at 90% polarization)^{#10}.

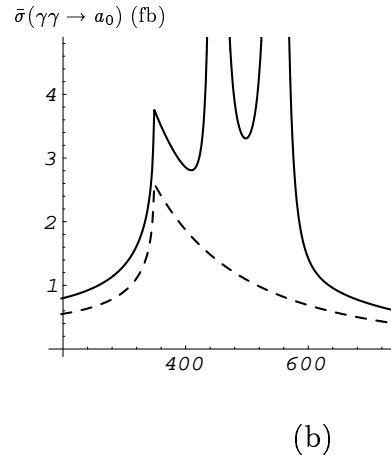
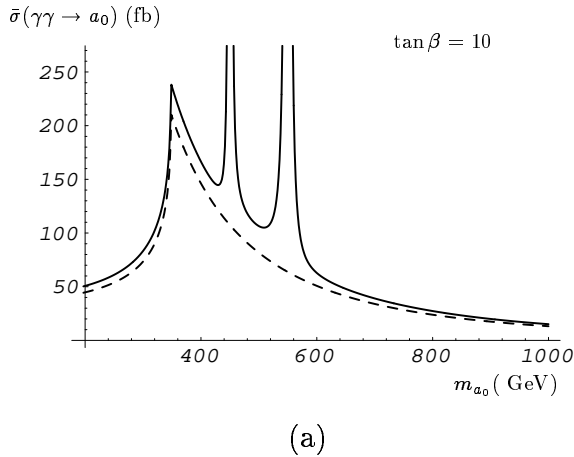


Figure 4. Total a_0 production cross section averaged over an energy bin equal to the total width of the a_0 . The solid curves correspond to the theory with effective operators for $\Lambda = 1$ TeV, the dashed curves give the pure two-doublet model results. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^\pm} = 500$ GeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The electron polarization is 70%.

^{#10} For a more significant reduction exquisite degrees of polarization are required: 99% for $(1 - \boldsymbol{\xi}_+^2)/(1 - \boldsymbol{\xi}_- \cdot \boldsymbol{\xi}_+) \simeq 0.25$

These results imply that, even though it is theoretically possible to eliminate the light-physics contribution to this cross section for any value of β , experimental constraints on the initial electron's degree of polarization prevent the realization of this possibility. The expression for the cross section is simply (in the center of mass system)

$$d\sigma = \frac{\pi}{2s m_{a_0}} |\mathcal{A}_{\gamma\gamma \rightarrow a_0}|^2 \delta(\sqrt{s} - m_{a_0}). \quad (3.5)$$

In figure 4 we plot the value of this expression averaged over the width of the a_0 , namely

$$\bar{\sigma} = \frac{\pi}{2m_{a_0}^3} \frac{|\mathcal{A}_{\gamma\gamma \rightarrow a_0}|^2}{\Gamma_{a_0}}. \quad (3.6)$$

To illustrate the significance of this result we consider the ratio of $\bar{\sigma}$ to the same quantity when $\Lambda \rightarrow \infty$. As can be seen from Fig. 5, the deviations are important for $\Lambda = 1$ TeV.

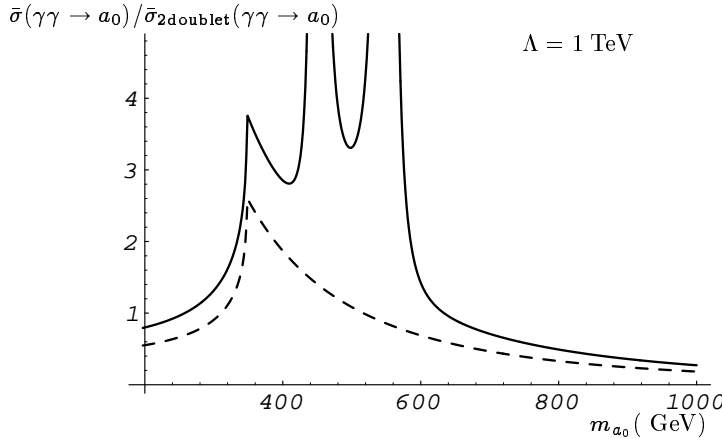


Figure 5. Ratio of the total to light cross sections. The solid curve correspond to $\tan\beta = 30$, the dashed curve to $\tan\beta = 10$. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^+} = 500$ GeV and $\Lambda = 1$ TeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The electron polarization is 70%.

The number of events to be generated at a $\gamma\gamma$ collider with 10/fb/yr luminosity

is presented in Fig. 6. As in the previous cases, deviations from the usual two-doublet model are noticeable for large values of $\tan\beta$ only.

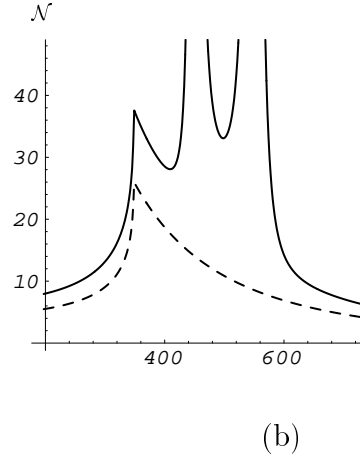
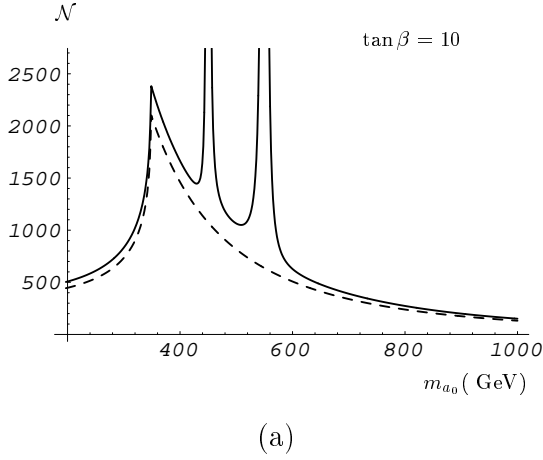


Figure 6. Number of a_0 events produced in $\gamma\gamma$ collisions for two values of $\tan\beta$. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{a_0} = m_{H^+} = 500$ GeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The electron polarization is 70% and the luminosity equals 10/fb.

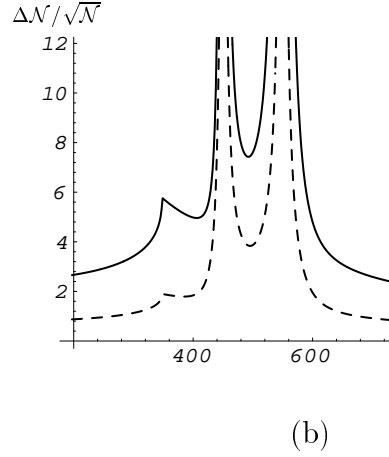
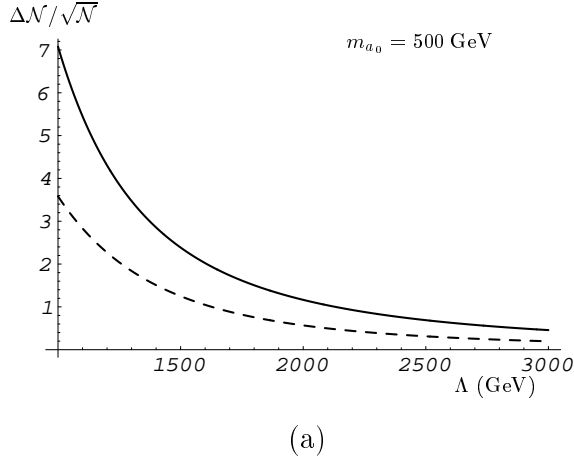


Figure 7. Statistical significance of the deviations from the light cross section, solid curve $\tan\beta = 30$, dashed curve $\tan\beta = 10$. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{a_0} = m_{H^+} = 500$ GeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The electron polarization is 70% and the luminosity equals 10/fb.

Finally, in order to estimate the statistical significance of the deviations from the light-physics result, we evaluate \mathcal{N} , the total number of events and $\Delta\mathcal{N}$, the

total number of events minus the number of events present when $\Lambda \rightarrow \infty$. Then $\Delta\mathcal{N}/\sqrt{\mathcal{N}}$ is a measure of the statistical significance of the deviations from the light-physics results and is plotted in Fig. 7 for a 10/fb luminosity. As can be seen from this plot there are significant deviations from the light-physics results up to a few TeV.

4. Conclusions

We have employed the method of effective lagrangians in evaluating the possible deviations from the two-doublet extension of the standard model for reactions containing the CP odd excitation a_0 and two photons. The calculations illustrate the fact that effective lagrangians can be used in loop calculations.

The magnitude of the resets can be estimated reliably within a set of general scenarios. For example, we must examine whether CP violation operators are generated at the same scale as the CP conserving ones, or whether the underlying physics is weakly or strongly coupled. One of the advantages of the effective lagrangian approach is the ease with which these various possibilities are identified and studied.

The cases which we studied in detail were the two-photon processes related to the CP-odd scalar a_0 . We found important deviations stemming from the anomalous couplings when the scale of new physics is moderate large (a few TeV) provided $\tan\beta \gtrsim 20$. There are several problems with the type of processes considered stemming mainly from the small branching ratios and cross sections. For the LHC, the event number is so reduced (see Fig. 2) that the observation of the effects generated by the physics beyond the two-doublet model is very unlikely (except for a 10-year study). In contrast, for a $\gamma\gamma$ collider with a luminosity of $\sim 10/\text{fb}$, a careful study will uncover some effects generated by the physics beyond the two doublet model, or at least place significant bounds on the scale of this type of new physics (see Figs. 6,7.).

It was mentioned in the introduction (as evident from the form of the potential) that no CP violation effects were included in the light theory. In contrast, several effective operators (such as $\mathcal{O}_{ijkl}^{(1,3)}$) violate CP. This situation is not inconsistent. In fact, CP violations in the potential can be pictured as $h_0 - a_0$ and $H_0 - a_0$ mixings of the same type as those induced by $\mathcal{O}^{(1,3)}$. Therefore the presence of CP violating terms in the potential will not affect the conclusions as long as their order of magnitude is the same as that of the mixing terms generated by the effective operators. More precisely, we have assumed that CP violation is generated at a scale $\gtrsim \Lambda$.

The numerical results were obtained not by the most optimistic choice of parameters, but by mimicking the possible presence of cancellations among various graphs. As noted in the appendix there is still the possibility of having further suppression factors, in which case the new physics effects will be too small to be observed. On the other hand there could be some enhancements, in which case our results would constitute a lower bound on the new physics effects.

The results presented in this paper are to be compared to the ones derived in the minimal Standard Model where the (CP-even) $H\gamma\gamma$ interaction is generated through one-loop effects of charged fermions and W gauge bosons. The contributions from the W boson and the top quark are dominant, although the latter is only marginally important and tends to cancel the first one partially. When the contribution of dimension six operators is considered, the virtual effects may enhance the Standard Model width of this rare decay mode up to one order of magnitude.^{14,15} Recently it was pointed out that tree-level generated bosonic operators of dimension eight may also enhance the Standard Model result.¹⁵

The conclusions of this paper are, by necessity, speculative. The a_0 has not been found as of yet and so the study of its decays lies in the future. Still, based on the above calculations it is clear that if this excitation is observed, the detailed study of its interactions with two photons may open a window into new physics.

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APPENDIX

In this appendix we present the full list of operators contributing to the $a_0\text{-}\gamma\text{-}\gamma$ three point function and the corresponding amplitudes.

The list of (tree-level-generated) operators which contribute at one loop is the following ($i, k, l, m, n = 1, 2$)

$$\begin{aligned}
\mathcal{O}_{e\varphi;ijk} &= \left(\phi_i^\dagger \phi_j\right) (\bar{\ell} e \phi_k); \\
\mathcal{O}_{d\varphi;ijk} &= \left(\phi_i^\dagger \phi_j\right) (\bar{q} d \phi_k); \\
\mathcal{O}_{u\varphi;ijk} &= \left(\phi_i^\dagger \phi_j\right) (\bar{q} u \tilde{\phi}_k); \\
\mathcal{O}_{\phi_k f} &= i \left(\phi_k^\dagger D_\mu \phi_k\right) (\bar{f} \gamma^\mu f); \quad f = e, u, d; \\
\mathcal{O}_{\phi_k F}^{(1)} &= i \left(\phi_k^\dagger D_\mu \phi_k\right) (\bar{F} \gamma^\mu F); \quad F = \ell, q; \\
\mathcal{O}_{\phi_k F}^{(3)} &= i \left(\phi_k^\dagger \tau^I D_\mu \phi_k\right) (\bar{\ell} \gamma^\mu \tau^I \ell); \quad F = \ell, q; \\
\mathcal{O}_{\varphi;ijklmn} &= \left(\phi_i^\dagger \phi_j\right) \left(\phi_k^\dagger \phi_l\right) \left(\phi_m^\dagger \phi_n\right); \\
\mathcal{O}_{\partial\varphi;ijkl} &= \frac{1}{2} \left[\partial_\mu \left(\phi_i^\dagger \phi_j\right)\right] \left[\partial_\mu \left(\phi_k^\dagger \phi_l\right)\right]; \\
\mathcal{O}_{ijkl}^{(1)} &= \left[\partial_\mu \left(\phi_i^\dagger \phi_j\right)\right] \left[\phi_k^\dagger \overleftrightarrow{D}_\mu \phi_l\right]; \\
\mathcal{O}_{ijkl}^{(3)} &= \left[\phi_i^\dagger D_\mu \phi_j\right] \left[(D^\mu \phi_k)^\dagger \phi_l\right];
\end{aligned} \tag{A.1}$$

where only operators consistent with the discrete symmetry are retained, for example $\mathcal{O}_{e\varphi;ijk}$ is considered only with $i+j+k$ odd, $\mathcal{O}_{\varphi;ijklmn}$ when $i+j+k+l+m+n$ even, etc. As mentioned above, we have adopted the notation of Ref. 9. It is worth

pointing out that the operator $\mathcal{O}_{ijkl}^{(1)}$ is slightly different from the one used in Eq. (3.14) of Ref.9, the difference between this expression and ours is easily seen to be, using the equations of motion, a linear combination of $\mathcal{O}_{\varphi;ijklm}$ and $\mathcal{O}_{\partial\varphi;ijkl}$; therefore both expressions are equivalent [16]. The one we chose enormously simplifies the calculation of the corresponding amplitude (reducing contributions from 19 to 3 graphs).

The operators $\mathcal{O}_{\varphi;ijklmn}$, $\mathcal{O}_{\partial\varphi;ijkl}$ and $\mathcal{O}_{ijkl}^{(1,3)}$, generate quadratic terms which mix a_0 with the scalars, h^0 and H^0 (this is a consequence of the fact that these operators violate CP). Therefore these operators produce two kind of graphs in the unitary gauge: 1PI diagrams with H^+ in the loop, and 1PR diagrams where the a_0 converts into a h^0 or H^0 via the above mixing terms, and then the h^0 or H^0 decay into two photons via H^+ , W or fermion loops. The amplitude corresponding to the above \mathcal{O} via 1PR graphs will be denoted by a “(1PR)” superscript (no 1PI superscript will be used in the other amplitudes in order to simplify the notation)

We denote by \mathcal{A}_i the value of the on-shell three point function generated by \mathcal{O}_i (without the factor y_i/Λ^2), straightforward evaluation of the relevant Feynman rules and graphs yields, for the 1PI contributions containing bosons in the loop, generated by $\mathcal{O}_{\varphi ijklm}$, $\mathcal{O}_{\partial\varphi ijkl}$ and $\mathcal{O}_{ijkl}^{(a)}$, ($a = 1, 3$) yield (f stands for the right-handed fermions $f = e, u, d$)

$$\left. \begin{aligned} \mathcal{A}_{\varphi kk1212} \\ \mathcal{A}_{\varphi kk2121} \end{aligned} \right\} &= \pm \frac{(-)^k m_w^3 s_w^3 s_{4\beta}}{4\pi^2 m_{a_0}^2 e} \left[2 \frac{m_{H^+}^2}{m_{a_0}^2} I_{-1}(m_{H^+}) + 1 \right] \mathcal{P}_1 \\ \left. \begin{aligned} \mathcal{A}_{\partial\varphi 1212} \\ \mathcal{A}_{\partial\varphi 2121} \end{aligned} \right\} &= \mp \frac{m_w s_w e s_{2\beta}}{8\pi^2} \left[2 \frac{m_{H^+}^2}{m_{a_0}^2} I_{-1}(m_{H^+}) + 1 \right] \mathcal{P}_1 \\ \mathcal{A}_{ijkl}^{(a)} &= \frac{e m_w s_w s_{2\beta} h_{ijkl}^{(a)}}{4\pi^2} \left[2 \frac{m_{H^+}^2}{m_{a_0}^2} I_{-1}(m_{H^+}) + 1 \right] \mathcal{P}_1; \quad a = 1, 3 \end{aligned} \quad (\text{A.2})$$

where m_{H^+} denotes the mass and

$$I_n(\mu) = \int_0^1 dx x^n \ln [1 - x(1-x)m_{a_0}^2/\mu^2]; \quad n \geq -1. \quad (\text{A.3})$$

The coefficients in (A.2) are

$$\begin{aligned}
h_{11kk}^{(1)} &= (-)^k s_\beta^2; & h_{22kk}^{(1)} &= (-)^k c_\beta^2; & h_{1212,1221,2112,2121}^{(1)} &= -\frac{1}{2} c_{2\beta}. \\
h_{2211,2121}^{(3)} &= +1; & h_{1122,1212}^{(3)} &= -1; & h_{1111,2222,1221,2112}^{(3)} &= 0.
\end{aligned} \tag{A.4}$$

The 1PI contributions with fermions in the loop are

$$\begin{aligned}
\mathcal{A}_{f\varphi ijk} &= \frac{s_\beta b_{ijk}^f m_w^2 s_w^2 Q_f^2 m_f}{2\sqrt{2} \pi^2 m_{a_0}^2} \left\{ \left[\left(\frac{4m_f^2}{m_{a_0}^2} - 1 \right) I_{-1}(m_f) + 2 \right] \mathcal{P}_1 \right. \\
&\quad \left. - I_{-1}(m_f) \mathcal{P}_2 \right\} \\
\mathcal{A}_{\phi_k f} &= \frac{e s_w m_w s_{2\beta} Q_f^2 (-)^k}{8\pi^2} \left[2 \frac{m_f^2}{m_{a_0}^2} I_{-1}(m_f) + 1 \right] \mathcal{P}_2 \\
\mathcal{A}_{\phi_k, \ell}^{(1,3)} &= \frac{e s_w m_w s_{2\beta} Q_f^2 (-)^{k+1}}{8\pi^2} \left[2 \frac{m_e^2}{m_{a_0}^2} I_{-1}(m_e) + 1 \right] \mathcal{P}_2 \\
\mathcal{A}_{\phi_k, q}^{(1,3)} &= \frac{e s_w m_w s_{2\beta} Q_f^2 (-)^{k+1}}{8\pi^2} \sum_{f=d,u} d_f^{(1,3)} \left[2 \frac{m_f^2}{m_{a_0}^2} I_{-1}(m_f) + 1 \right] \mathcal{P}_2
\end{aligned} \tag{A.5}$$

where

$$\begin{aligned}
b_{111,221,212,122}^{e,d} &= -c_\beta^2, -s_\beta^2, -s_\beta^2, 1 + c_\beta^2; & b_{112,121,211}^u &= c_\beta^2, c_\beta^2, -1 - s_\beta^2 \\
d_f^{(a)} &= \begin{cases} -1 & \text{for } a = 3, f = u \\ +1 & \text{otherwise} \end{cases}
\end{aligned} \tag{A.6}$$

Finally the 1PR contributions are generated, as mentioned above, by the contact terms in some operators which induce $a_0 - h^0$ and $a_0 - H^0$ mixings, the h^0 and H^0 then decay into two photons via fermion, W or H^\pm loops (in the unitary

gauge). The results can be extracted from [17, 10], the amplitude is ($\phi = h^0, H^0$)

$$\begin{aligned} \mathcal{A}_\phi^{(1PR)}(\mathcal{O}_i) = & \frac{\eta_i(\phi)}{m_{a_0}^2 - m_\phi^2} \frac{es_w m_w^3}{2\pi^2} \left\{ u_{\phi H} \left[1 + \frac{2m_{H^+}^2}{m_{a_0}^2} I_{-1}(m_{H^+}) \right] \right. \\ & + 3u_{\phi W} \left\{ \left(\frac{m_{a_0}^2}{2m_w^2} - 1 \right) \left[1 + \frac{2m_w^2}{m_{a_0}^2} I_{-1}(m_w) \right] + \frac{1}{2} \right\} \\ & \left. + \sum_f Q_f^2 N_f \frac{m_f^2 u_{\phi f}}{4m_w^2} \left[\left(1 - \frac{4m_f^2}{m_{a_0}^2} \right) I_{-1}(m_f) - 2 \right] \right\} \mathcal{P}_1 \end{aligned} \quad (\text{A.7})$$

where N_f denotes the number of colors for fermion f . The non-vanishing constants $\eta_i(\phi)$ are

$$\eta(h^0)_{11kk}^{(1)} = (-)^{k+1} c_\beta^2 s_\beta s_\alpha; \quad \eta(h^0)_{22kk}^{(1)} = (-)^k s_\beta^2 c_\beta c_\alpha;$$

$$\eta(h^0)_{1212,2121}^{(1)} = \frac{3c_{\alpha-\beta} + c_{\alpha+3\beta}}{4}; \quad \eta(h^0)_{1221,2112}^{(1)} = -s_\beta c_\beta s_{\alpha+\beta}$$

$$\eta(H^0)_{11kk}^{(1)} = (-)^k c_\beta^2 s_\beta c_\alpha; \quad \eta(H^0)_{22kk}^{(1)} = (-)^k s_\beta^2 c_\beta s_\alpha;$$

$$\eta(H^0)_{1212,2121}^{(1)} = \frac{3s_{\alpha-\beta} + s_{\alpha+3\beta}}{4}; \quad \eta(H^0)_{1221,2112}^{(1)} = s_\beta c_\beta c_{\alpha+\beta}$$

$$\eta(h^0)_{1122,2121}^{(3)} = -\eta(h^0)_{2211,1212}^{(3)} = \frac{1}{4} s_{2\beta} s_{\alpha-\beta}$$

$$\eta(H^0)_{1122,2121}^{(3)} = -\eta(H^0)_{2211,1212}^{(3)} = -\frac{1}{4} s_{2\beta} c_{\alpha-\beta}$$

$$\eta(h^0)_{\partial\varphi;1212} = -\eta(h^0)_{\partial\varphi;2121} = \frac{3}{2} c_{\alpha-\beta} - \frac{1}{2} c_{2\beta} c_{\alpha-\beta}$$

$$\eta(H^0)_{\partial\varphi;1212} = -\eta(H^0)_{\partial\varphi;2121} = \frac{3}{2} s_{\alpha-\beta} - \frac{1}{2} c_{2\beta} s_{\alpha-\beta}$$

$$\begin{aligned}
\eta(h^0)_{\varphi;kk1212} &= -\eta(h^0)_{\varphi;kk2121} = \frac{s_w^2 m_w^2 (1 - (-)^k c_{2\beta})}{4\pi\alpha m_{a_0}^2} \left[\frac{5}{2} c_{\alpha+\beta} \right. \\
&\quad \left. + c_{\alpha-\beta} \left((-1)^k - \frac{1}{2} c_{2\beta} \right) \right] \\
\eta(H^0)_{\varphi;kk1212} &= -\eta(H^0)_{\varphi;kk2121} = \frac{s_w^2 m_w^2 (1 - (-)^k c_{2\beta})}{4\pi\alpha m_{a_0}^2} \left[\frac{5}{2} s_{\alpha+\beta} \right. \\
&\quad \left. - s_{\alpha-\beta} \left((-1)^k - \frac{1}{2} c_{2\beta} \right) \right]
\end{aligned} \tag{A.8}$$

and

$$\begin{array}{ccccc}
& u_{\phi H} & u_{\phi W} & u_{\phi u} & u_{\phi d} \\
(\phi = h^0) & -s_{\alpha-\beta} + \frac{c_{2\beta} s_{\alpha+\beta}}{2c_w^2} & -s_{\alpha-\beta} & c_{\alpha}/s_{\beta} & -s_{\alpha}/c_{\beta} \\
(\phi = H^0) & c_{\alpha-\beta} - \frac{c_{2\beta} c_{\alpha+\beta}}{2c_w^2} & c_{\alpha-\beta} & s_{\alpha}/s_{\beta} & c_{\alpha}/c_{\beta}
\end{array} \tag{A.9}$$

The total contribution is then

$$\mathcal{A} = \mathcal{A}_{\text{standard}} + \frac{1}{\Lambda^2} \left(\sum_i y_i \mathcal{A}_i + \sum_i y_i \mathcal{A}_i^{(1PR)} \right) \tag{A.10}$$

where the constants y_i contain all couplings from the underlying theory which generates the operators \mathcal{O}_i . From this expression the decay width of the a_0 or its production cross section can be evaluated directly. Note that the terms containing \mathcal{P}_1 and \mathcal{P}_2 will not interfere in polarization averaged widths or cross-sections.

It must be kept in mind that all the coefficients will contain, in general, some coupling constants of the underlying theory which are small ($\lesssim 1$) by our assumption of it being a weakly coupled theory. For illustrative purposes we will choose $|y_i| = 1$ and we will neglect all anomalous contributions which are proportional to a fermion mass other than the top. In order to simulate possible cancellations between the various contributions of the same type (produces by constants y_i of opposite signs) we will use an averaging procedure such as

$$\sum_{ijk=112,121,211} y_{t\varphi;ijk} \mathcal{A}_{t\varphi;ijk} \rightarrow \frac{1}{3} \sum_{ijk=112,121,211} \mathcal{A}_{t\varphi;ijk} \tag{A.11}$$

This simple procedure implies several cancellations, for example, the contributions

from $\mathcal{A}_{\phi_k f}$ and $\mathcal{A}_{\phi_k f}^{(1,3)}$ cancel.

We emphasize that this example is presented for illustrative purposes only, nonetheless we expect the results to provide good semi-quantitative estimates. Should there be fewer cancellations the anomalous signal would be enhanced; in this case the results presented constitute a lower bound. On the other hand it is possible for the constants y_i could be suppressed by unknown effects in which case our results would over-estimate the anomalous effects.

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FIGURE CAPTIONS

- 1) Width and branching ratios for the decay $a_0 \rightarrow \gamma\gamma$ as a function of m_{a_0} for two values of β . The solid curves correspond to the theory with effective operators for $\Lambda = 1$ TeV, the dashed curves give the pure two-doublet model results. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^+} = 500$ GeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The resonance peaks are produced by the 1PR diagrams.
- 2) **(a),(b)**: Ratio of the total $a_0 \rightarrow \gamma\gamma$ decay width to the decay width of a Standard Model Higgs particle of the same mass into two photons. **(c),(d)** Total number of yearly events at the LHC (C.M. energy= 7TeV, luminosity 100/fb). The solid curves correspond to the theory with effective operators for $\Lambda = 1$ TeV, the dashed curves give the pure two-doublet model results. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^+} = 500$ GeV and $\Lambda = 1$ TeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$.
- 3) Branching ratio for the decay $a_0 \rightarrow \gamma\gamma$ for $\Lambda = 1$ TeV (solid curve), $\Lambda = 2$ TeV (dashed curve) and $\Lambda = \infty$ (dotted curve). We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^+} = 500$ GeV and $\tan\beta = 30$; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$.
- 4) Total a_0 production cross section averaged over an energy bin equal to the total width of the a_0 . The solid curves correspond to the theory with effective operators for $\Lambda = 1$ TeV, the dashed curves give the pure two-doublet model results. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^+} = 500$ GeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The electron polarization is 70%.
- 5) Ratio of the total to light cross sections. The solid curve correspond to $\tan\beta = 30$, the dashed curve to $\tan\beta = 10$. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{H^+} = 500$ GeV and $\Lambda = 1$ TeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The electron polarization is 70% .
- 6) Number of a_0 events produced in $\gamma\gamma$ collisions for two values of $\tan\beta$. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{a_0} = m_{H^+} = 500$ GeV; $m_{\text{top}} =$

174 GeV, $\alpha = 45^\circ$. The electron polarization is 70% and the luminosity equals 10/fb.

- 7) Statistical significance of the deviations from the light cross section, solid curve $\tan\beta = 30$, dashed curve $\tan\beta = 10$. We chose $m_{h_0} = 450$ GeV, $m_{H_0} = 550$ GeV, $m_{a_0} = m_{H^+} = 500$ GeV; $m_{\text{top}} = 174$ GeV, $\alpha = 45^\circ$. The electron polarization is 70% and the luminosity equals 10/fb.